Topic: Vertical motion, coin dropped from the roof

Question: Find instantaneous velocity.

A pumpkin is dropped from the top of a building and falls 5 m to the ground. Given the position function of the pumpkin, find instantaneous velocity at $t = 3$ seconds.

$$s(t) = -6t^2 + 3t - 5$$

Answer choices:

A -40 m/s

B -39 m/s

C -33 m/s

D -50 m/s

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Solution: C

To find the instantaneous velocity of an object in vertical motion we'll first take the derivative of the position function. Since velocity is the derivative of position, this will give us the velocity function, which models velocity at all time.

$$s(t) = -6t^2 + 3t - 5$$

$$v(t) = s'(t) = -12t + 3$$

Now that we have a velocity function that models velocity at all time, we can evaluate it at the given time to get velocity at that particular instant, which we call instantaneous velocity at $t = 3$.

$$v(3) = -12(3) + 3$$

$$v(3) = -33$$

The instantaneous velocity at $t = 3$ is -33 m/s. Because the velocity is negative, it means that the pumpkin is falling toward the ground.

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Topic: Vertical motion, coin dropped from the roof

Question: Find average velocity.

A baseball is dropped from the top of a bridge that's 8 m high. Given the baseball's position function, find the average velocity of the baseball during the first 4 seconds.

$$s(t) = -8t^2 - 4t - 8$$

Answer choices:

A -36 m/s

B -68 m/s

C -86 m/s

D -23 m/s

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Solution: A

The average velocity of an object is given by

$$v_{avg} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

Since we want to find average velocity during the first four seconds, we know that $t_1 = 0$ and $t_2 = 4$. Plugging these into our formula for average velocity, we get

$$v_{avg} = \frac{s(4) - s(0)}{4 - 0}$$

$$v_{avg} = \frac{s(4) - s(0)}{4}$$

We need to find $s(0)$ and $s(4)$ so that we can plug them into our formula.

$$s(0) = -8(0)^2 - 4(0) - 8$$

$$s(0) = -8$$

and

$$s(4) = -8(4)^2 - 4(4) - 8$$

$$s(4) = -152$$

Plugging these values into the average velocity formula above, we get

$$v_{avg} = \frac{s(4) - s(0)}{4}$$

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$$v_{avg} = \frac{-152 - (-8)}{4}$$

$$v_{avg} = \frac{-144}{4}$$

$$v_{avg} = -36$$

The average velocity during the first 4 seconds is -36 m/s. Because the velocity is negative, it means that the baseball is falling toward the ground.

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Topic: Vertical motion, coin dropped from the roof

Question: A coin is dropped from the roof of a 400 ft building with an initial velocity of -64 ft/s. When does it hit the ground and what is the velocity at that point?

Answer choices:

A The time when the coin hits the ground is 7.78 s and the velocity when the coin hits the ground is -108.48 ft/s.

B The time when the coin hits the ground is 3.39 s and the velocity when the coin hits the ground is -108.48 ft/s.

C The time when the coin hits the ground is 3.39 s and the velocity when the coin hits the ground is -172.48 ft/s.

D The time when the coin hits the ground is 7.78 s and the velocity when the coin hits the ground is -172.48 ft/s.

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Solution: C

To solve a vertical motion problem where a coin is dropped from the roof of a building, use the formula

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

where $y(t)$ is the position of the coin at time t, g is the gravitational constant, t is the time, v_0 is the initial velocity and y_0 is the initial position. In this type of problem, $t_0 = 0$ is the initial time.

To solve, start by using $g = 32$ ft/s², $v_0 = -64$ ft/s, and $y_0 = 400$ to create the equation that represents the motion of the ball.

$$y(t) = \frac{1}{2}gt^2 - v_0t - y_0$$

$$y(t) = \frac{1}{2}(32)t^2 - (-64)t - 400$$

$$y(t) = 16t^2 + 64t - 400$$

$$y(t) = 16\left(t^2 + 4t - 25\right)$$

The question is looking for the final time and the final velocity as the coin hits the ground. When the coin hits the ground, the height will be 0, which means we need to solve

$$0 = 16\left(t^2 + 4t - 25\right)$$

$$0 = t^2 + 4t - 25$$

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We'll use the quadratic formula to find the roots of the function.

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-25)}}{2(1)}$$

$$t = \frac{-4 \pm \sqrt{16 + 100}}{2}$$

$$t = \frac{-4 \pm 2\sqrt{29}}{2}$$

$$t = -2 \pm \sqrt{29}$$

This gives us two roots: $t = -2 + \sqrt{29}$ and $t = -2 - \sqrt{29}$. Time t can't be negative, which means that the coin hits the ground when $t = -2 + \sqrt{29} \approx 3.39$.

Because velocity is the derivative of position, to find velocity when the coin hits the ground, we'll take the derivative of the function we found for $y(t)$.

$$y(t) = 16t^2 + 64t - 400$$

$$y'(t) = v(t) = 32t + 64$$

Plug in $t \approx 3.39$ to find velocity.

$$v(3.39) = 32(3.39) + 64$$

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$$v(3.39) = 108.48 + 64$$

$$v(3.39) = 172.48$$

But the velocity is negative since the coin is falling when it hits the ground, so we'll say $v(3.39) = -172.48$.

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